XI The toolbox

In this chapter, I will collect some remarks on economic modelling that will be used in various instances throughout part Three of this book. First, I will offer some general remarks on how models are used to arrive at theoretical predictions. Second, I will turn to microeconomic concepts, in particular the model of person-to-person exchange (named after Edgeworth), the model of impersonal exchange (provided by Walras), and noncooperative game theory. Leaving microeconomics aside, we will then turn to cooperative game theory and, in particular, the Shapley value.

A Models and theoretical predictions

Economic theory-building procedes in three steps:

1. A model is described. It is meant to reproduce important aspects of reality. But, of course, it is only a very simplified mirror of reality "out there".
2. A theoretical prediction of "what will happen" is produced. What are the strategies chosen by the agents? What prices will prevail? What are the players' payoffs? The theoretical predictions are derived by applying so-called solution concepts, such as the "best" decision, the Nash equilibrium, the Walras equilibrium, the Shapley value, and so forth.
3. Finally, one can ask the question of how the theoretical predictions (variables, outcomes) depend on the model itself (parameters, data, input).

Readers might often object to particular modelling strategies. In particular, they may feel that a given model oversimplifies the giving or gifting situation in question. There are two possible responses to such objections. Firstly, simplifications serve the useful purpose of concentrating on the most important aspects of the modelled situation. Secondly, one may build a more detailed model if one thinks that additional details are vital in order to understand hitherto unexplored, and yet relevant, issues.

B Person-to-person (Edgeworthian) exchange

(1) Introduction

Allocation of goods takes place in two different modes—the first of these being person-to-person. The second mode is impersonal trading, expounded by General Equilibrium Theory (see the next section). A key message is that trade in both modes may benefit all parties involved. A second message, beloved by many economists, is the following: Free markets are wonderful.

(2) Pareto improvements

Exchange (of goods—in a wide sense) can be beneficial to all parties involved. This idea is closely related to the concept of "Pareto[624] improvement". Situation 1 is deemed Pareto superior in relation to another situation 2 if no individual is worse off in the first than in the second, while at least one individual is strictly better off. Then, the move from situation 2 to situation 1 is called a Pareto improvement. Situations are referred to as Pareto-efficient, Pareto-optimal, or simply efficient if Pareto improvements are not possible.

Economists often assume that bargaining leads to an efficient outcome under ideal conditions. As long as Pareto improvements are available, one could argue that there is no reason not to "cash in" on them.[625]

(3) Matching models

A particular type of Edgeworthian model are matching models. Here, the "goods" to be exchanged are the people themselves, who engage in the process of exchanging. Marriages (between prospective brides and grooms) or internships (of medical students in hospitals) provide suitable examples.[626] *Kanyādāna* is covered in chapter XIV.

624 Vilfredo Pareto, Italian sociologist, 1848–1923

625 However, the existence of Pareto improvements does not make their realisation a foregone conclusion. This is obvious from the famous prisoners' dilemma (see, for example, Gibbons (1992, pp. 2–5)). See the game-theory section in this chapter.

626 See the eminently readable book by Roth (2016). Alvin Roth is *the* pioneer in the field of matching economics. He obtained the Nobel prize in Economic Sciences in 2012.

C Impersonal (Walrasian) exchange

The impersonal-trading mode is formalised in General Equilibrium Theory (GET). Here, the agents are confronted with market prices. At these prices, they choose (what are for them) the optimal amounts of
(i) labour they wish to offer (households) or demand (firms) on the labour market
(ii) goods they wish to sell (firms) or buy (households).
None of these agents buy or sell from any particular person, but rather anonymously "on the market". At the prevailing prices, they are imagined to be free to buy or sell as many units as they like.

One may imagine that the prices are taken as given in the short run. However, at some price constellations, demand may be greater than supply for some particular goods. Then, one might expect that prices for these goods will be driven upwards. Inversely, prices may go down if supply exceeds demand. In the long run, one may expect prices that equalise demand and supply. While this dynamic perspective (short run, long run, price adaptations) is not modelled explicitly in GET, it nevertheless underlies the rationale of this model.

The aim of GET is to find (or to establish the existence of) a so-called Walras equilibrium, where

[IR] all actors behave in a utility-[627], or profit-maximising manner, and
[DS] all the buying and selling decisions can be carried out.

Here, IR stands for "individual rationality" and DS for "demand equals supply".

In general, a Walras equilibrium can be defined for many goods and many agents. Thus, one obtains a model of a decentralised market system where individual producers and consumers make their buying and selling decisions on the basis of given prices. One theoretical question is whether one can be certain that prices exist for all goods such that the two conditions of individual optimisation and equality of demand and supply are fulfilled. Under certain assumptions, this "existence" question can be answered affirmatively.[628] Under more stringent conditions, there exists exactly one such Walras equilibrium.

General Equilibrium Theory is also concerned with the relationship between the Pareto efficient outcomes in a person-to-person exchange model (see section B) and the equilibrium outcomes in a model of impersonal exchange. Under rather general conditions, equilibria in GET are found to be Pareto efficient. This is the so-called First Welfare Theorem. It can be considered a formal expresssion of Adam Smith's "invisible hand". If one thinks that Pareto efficiency is a good thing, then, indeed, free markets are wonderful.

627 I do not discuss the intricate concept of "utility" in this book. The interested reader can refer to any microeconomic textbook. I use "utility" and "payoff" interchangeably.
628 See Hildenbrand & Kirman (1988).

Leaving aside Pareto efficiency, there is a second, perhaps even more relevant argument for free markets and prices. Going beyond (basically) static General Equilibrium Theory, one may follow the Nobel-prize winner (in Economic Sciences, 1974) Friedrich-August von Hayek. One of his research interests concerns the question of how people obtain information in order to make good decisions. Since society needs to adapt to constant changes, Hayek (1945, p. 524) insists on decentral decisions "because only thus can we ensure that the knowledge of the particular circumstances of time and place will be promptly used. But the 'man on the spot' cannot decide solely on the basis of his limited but intimate knowledge of the facts of his immediate surroundings. There still remains the problem of communicating to him such further information as he needs to fit his decisions into the whole pattern of changes of the larger economic system."

According to Hayek (1945, p. 526), it is the prices that coordinate actions of people: "Assume that somewhere in the world a new opportunity for the use of some raw material, say tin, has arisen, or that one of the sources of supply of tin has been eliminated. It does not matter for our purpose—and it is very significant that it does not matter—which of these two causes has made tin more scare. All that the users of tin need to know is that some of the tin they used to consume is now more profitably employed elsewhere, and that in consequence they must economize tin."

Thus, the increase of tin prices induces people to come to terms with the scarcity of tin. For Hayek (1945, p. 527), the price system is "a kind of machinery for registering change". He goes on to say: "The marvel is that in a case like that of a scarcity of one raw material, without an order being issued, without more than perhaps a handful of people knowing the cause, tens of thousands of people whose identity could not be ascertained by months of investigation, are made to use the material or its products more sparingly, i.e., they move in the right direction."

D Noncooperative game theory

Game theory presupposes a set of players—usually at least two. Noncooperative game theory belongs to the realm of microeconomics. The players have either strategies or actions at their disposal and try to maximise their payoffs. In contrast, there are no explicit actions or strategies in cooperative game theory. Section XI.E deals with the Shapley value as arguably the most important concept from cooperative game theory.

(1) Strategic games

In strategic games, the players each simultaneously choose a strategy and obtain a payoff that depends on the strategy combination, i.e., on the tuple of strategies chosen

Table 5: A strategic game

		Player 2	
		left	right
Player 1	up	$(4, 5)$	$(6, 0)$
	down	$(3, 1)$	$(2, 7)$

by all players. This is the topic of this (first) subsection. In the next subsection, sequential games are dealt with. In these games, players choose actions in some prespecified order.

Consider the strategic game of Table 5. Player 1 has the two strategies "up" and "down", player 2 can choose between "left" and "right". If player 1 chooses up and player 2 chooses right, player 1 obtains a payoff of 6, while player 2 receives 0. That is, the first number indicates the payoff for player 1 and the second number is the payoff for player 2. Strategy tuples such as (up, right) are called strategy combinations.

Within the realm of strategic games, the two main solution concepts are "dominant strategy" and "Nash equilibrium".[629] A dominant strategy is a best strategy irrespective of the other players' strategies. In our strategic game, up dominates down because of the two inequalities $4 > 3$ and $6 > 2$. Player 2 does not avail of a dominant strategy. If a player has a dominant strategy, he can safely disregard the other players. Whatever they may choose, he himself cannot do any better than choosing the dominant strategy.

If a dominant strategy does not exist for all players, the concept of a Nash equilibrium might be employed. A Nash equilibrium is a strategy combination such that no player can profit from deviating unilaterally. Differently put, given that the other players stick to their respective strategies, each player chooses a best strategy. Thus, the Nash equilibrium imposes a specific kind of stability. The strategy combination (up, left) is a Nash equilibrium by virtue of $4 \geq 3$ and $5 \geq 0$.

(2) Sequential games

Consider the sequential game between the players 1 and 2 depicted in Figure 1. Some nodes are indexed by the player names (1 or 2). At these nodes, player 1 or 2 has to make a choice. Player 1 moves first, at the initial node (the leftmost node), choosing up or down. Next, it is player 2's turn, choosing between left and right. When both players have chosen their actions, they obtain the corresponding payoffs or "utilities". The payoff information is noted near the terminal nodes (the rightmost nodes).

Backward induction means "looking ahead" by "proceeding backwards". Before player 1 can decide on his move, he needs to know how player 2 will react to up, or

629 For example, see Gibbons (1992, pp. 1–12).

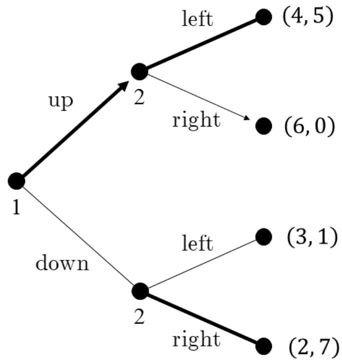

Figure 1: A game tree

down, chosen by player 1. Thus, backward induction starts with the players that move last. Consider the node where player 2 has to make a decision after player 1 chose up. Comparing the payoffs 5 and 0, player 2 chooses left. The edge that corresponds to the action left has been reinforced. In contrast, player 2 will choose right if he learns that player 1 has chosen down (this follows from $7 > 1$).

Now, after knowing the choices of player 2, we can look at player 1's decision. If he chooses up, player 2 will choose left, making it so that player 1 obtains a payoff of 4. If, however, player 1 chooses down, player 2 will choose right, making it so that player 1 obtains 2. Comparing 4 and 2, it is obvious that player 1 should, or will, choose up.

Thus, player 1 choosing up and player 2 choosing left is the predicted outcome. However, this may not be the observed outcome. For example, player 1 choosing up and player 2 choosing right is indicated by the arrows. In that sequence of actions, player 2 would have made a mistake. By $5 > 0$ he could have done better.[630]

E Shapley value[631]

(1) Cooperative game theory

The Shapley value belongs to the realm of cooperative game theory.[632] This theory presupposes n players that are collected in a set $N = \{1, 2, \ldots, n\}$, and a so-called coalition function v. A subset K of N is also called a coalition. N itself is called the grand coalition. To each coalition K, the coalition function attributes a "worth" $v(K)$.

630 See Wiese (2012), who argues that the idea of backward induction was already present in some Old Indian fables.

631 This section borrows freely from Wiese (2009, 2021, 2022b).

632 See Shapley (1953) for the ground-breaking contribution of the Nobel-prize winner (in Economic Sciences, 2012) Lloyd Shapley. Driessen (1988) is a textbook treatment of cooperative game theory.

The worths stands for the economic, social, political, or other gain that the particular group of players can achieve. A worth can only be created if at least one player is present, i.e., the empty set $\emptyset$ creates the worth zero, $v(\emptyset) = 0$. For ease of notation, one can write $v(i)$ instead of $v(\{i\})$, $v(1,2)$ instead of $v(\{1,2\})$, and $v(K \cup i)$ instead of $v(K \cup \{i\})$.

The aim of cooperative game theory is to specify payoffs for the players. These payoffs depend on the coalition function. Assume just two players, 1 and 2. A solution function φ defines the payoffs $\varphi_1(v)$ and $\varphi_2(v)$ for each coalition function v.

Cooperative game theory uses two different approaches to arrive at payoff vectors from coalition functions. (i) The algorithmic approach applies some algebraic manipulations to the coalition functions in order to derive payoff vectors. For example, each player might obtain the worth of his one-man coalition plus 5. This solution function would be described by $\varphi_1(v) = v(1) + 5$ and $\varphi_2(v) = v(2) + 5$. (ii) The axiomatic approach suggests general rules of distribution. One axiom might stipulate that the worth of the grand coalition $\{1,2\}$ is distributed among the players: $\varphi_1(v) + \varphi_2(v) = v(1,2)$. A second axiom might demand payoff equality. These two axioms together define a specific solution function, namely the one given by $\varphi_1(v) = \varphi_2(v) = \frac{v(1,2)}{2}$.

(2) The algorithmic approach

The Shapley value's algorithm builds on the players' "marginal contributions". A player's marginal contribution is the worth of a coalition with him minus the worth of said coalition without him, i.e., the difference he makes. In the two-player case, player 1 has two marginal contributions, the first with respect to the empty set $\emptyset$ (the marginal contribution is $v(1) - v(\emptyset)$), the second with respect to $\{2\}$ (with marginal contribution $v(1,2) - v(2)$).

Player 1's Shapley value is the average of his marginal contributions, taken over all sequences (rank orders) of the two players. For two players, there are just two sequences: player 1 may be first (sequence $(1,2)$) or second (sequence $(2,1)$). Thus, the players' Shapley values are

[1] $$Sh_1 = \frac{1}{2}(v(1) - v(\emptyset)) + \frac{1}{2}(v(1,2) - v(2))$$

and

[2] $$Sh_2 = \frac{1}{2}(v(2) - v(\emptyset)) + \frac{1}{2}(v(1,2) - v(1))$$

(3) The axiomatic approach

For any number of players and any coalition function, the Shapley value fulfils these axioms:

- The sum of the Shapley values equals the worth of the grand coalition, i.e., efficiency: $Sh_1 + Sh_2 = v(1,2)$
  in the case of two players. This property means that the grand coalition forms and the Shapley value distributes the worth of the grand coalition among the players.
- If a player 1 withdraws[633] from the game, another player 2's damage in terms of his Shapley payoff is equal to the damage that player 1 endures should player 2 withdraw, i.e.,
  withdrawal symmetry: $Sh_2 - v(2) = Sh_1 - v(1)$
  in the case of two players. Consider the left side of the equation. If player 1 withdraws, player 2 does not obtain the Shapley value Sh_2 anymore, but the Shapley value of the game of which he is the only player. In that game, he obtains the worth $v(2)$ of his one-man coalition. This is clear from the only rank order that exists in that game, as well as from the efficiency property.

These axioms of efficiency and withdrawal symmetry lead to the Shapley values in equations [1] and [2] above. Cooperative game theorists therefore say that these axioms axiomatise the Shapley value. This means that the Shapley value (in its algorithmic form, see subsection (2)) fulfils these axioms, and that there is no value different from the Shapley value which also obeys these axioms. This particular axiomatisation is provided by the Nobel-prize winner (in Economic Sciences, 2007) Roger Myerson (1980).

(4) Withdrawal symmetry and balancedness

Consider two examples of withdrawal symmetry. The first one originates with the sociologist Emerson (1962). Imagine two children A and B that often play together. Since they differ in their preferences, they take turns in playing their respective favourite games. In that situation, says Emerson, power-over is balanced as one might expect from withdrawal symmetry. Now, assume that child B in the A-B relationship finds another playing buddy C. Then, power-over is unbalanced. A would suffer more if B decides to no longer play with A than the other way around. After all, B can turn to her newfound alternative C. In that situation, argues Emerson, balancing operations set in that lead to B imposing her favourite game on A more often than before. From the point of view of the Shapley value (that was not known to Emerson), the effect of that balancing operation is to restore withdrawal symmetry.

The second example concerns a market where one seller S confronts four potential buyers B1 through B4. The object that S possesses has no value for him, but if any of the buyers manages to obtain this object, a worth of 1 is created. It can be shown that S obtains the Shapley value of $\frac{4}{5}$ in this game with four potential buyers, but only

633 Withdrawal means that the player set is reduced by the withdrawing player(s) and that the worths for the remaining players remain the same.

the Shapley value of $\frac{3}{4}$ in another game with only three potential buyers. Thus, the seller does not suffer a lot (only by $\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$) if buyer B1 withdraws. Consider now the change in buyer B1's Shapley value should the seller withdraw. Without the seller, B1's Shapley value is zero. In the presence of the seller, B1 will obtain the object with the same probability as any buyer: $\frac{1}{4}$. The seller's payoff $\frac{4}{5}$ can be understood as the price the successful buyer has to pay to the seller. Since the worth of the object in the hand of buyer B1 is 1, that buyer's Shapley value is $\frac{1}{4} \cdot \left(1 - \frac{4}{5}\right) = \frac{1}{20}$. Thus, withdrawal symmetry holds. The balancing operations consist of the low probability of obtaining the object together with the relatively high price.

Wiese (2021, 2022b) interprets withdrawal symmetry as "balancedness". The concept of "balance" developed by Emerson has been addressed by Blau (1964, p. 118: fn. 7), who considers it "somewhat confusing inasmuch as it diverts attention from the analysis of power imbalance". The obvious way out of this confusion is a distinction between the short run and the long run. In the short run, power differentials can exist, but they are diminished in the long run by balancing operations. From that perspective, balancedness becomes a very plausible and useful working tool.

The reason for stressing withdrawal symmetry in this book will become clear in section XIV.C on a puzzle observed by Parry and in section XVI.D, where *bali* taken by kings is explained in the context of the contest between the vital functions for superiority. Furthermore, remember Trautmann 's (1981, p. 285) "conundrum" about the conflict between spiritual and worldly power. Thapar (2013, p. 134) opines: "The ranking order between *brāhmaṇa* and *kṣatriya* is ambivalent to begin with where the former is dependent on the latter for *dāna* and *dakṣiṇā* and the latter requires that his power be legitimized by the former." From the point of view of balancedness, this assessment seems reasonable.

(5) Negative sanctions

One would be mistaken in thinking that the Shapley value only works for economic and social exchanges, but not for threats or extortions. Consider a threat uttered by a player 1 intent on armed robbery, as in ⟨149⟩. Even with a gun pointing to the head of player 2 (the victim), withdrawal symmetry still holds. It is important to note that withdrawing is analysed within the given game. The question of whether a player can quit the game or opt out is a totally different one. In market games, withdrawal simply means "not buying" or "not selling". In games with negative sanctions, withdrawal means not to give in to the threat. This does not mean that the robber and his gun mysteriously disappear.

The corresponding coalition function might obey $v(1,2) = 0$. If player 2 hands over the amount of money D to player 1, the robber's gain is the victim's loss. One then finds $Sh_1 = D$ and $Sh_2 = -D$. The efficiency axiom is fulfilled.

One might be tempted to set $v(2) = 0$, as the victim (player 2) does not lose any money if the robber withdraws. However, what the victim can achieve still depends on what the robber is doing (withdrawal is not quitting). If player 2 does not hand over the money peacefully, the robber may resort to violence, causing injury to the victim. Let i stand for the pain of being injured. Thus, one finds $v(2) = -i < 0$. Similarly, if player 2 runs away, the robber may injure the victim. Then, the robber will be in fear of prosecution for causing injury. Let f stand for this fear so that one obtains $v(1) = -f < 0$.

In the present case, withdrawal symmetry means

[3] $$-D - (-i) = Sh_2 - v(2) = Sh_1 - v(1) = D - (-f)$$

This equality can be used to calculate D, the amount of money handed over to the robber. It is given by

[4] $$D = \frac{i - f}{2}$$

The smaller the robber's fear of prosecution and the larger the victim's fear of injury, the greater the robber's loot.